EUREKA

THE JOURNAL OF THE ARCHIMEDEANS

(The Cambridge University Mathematical Society; Junior Branch of the Mathematical Association)

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Editorial

MODERN thought is populated by a race of anaemic abstractions. These are the Average Men, that great army of representative human beings. The oldest of these is the Law's Reasonable Man, pursuing his cautious and untrusting path through life from time immemorial. He has now been joined by the Economic Man, the Man-in-the-Street, and a host of others.

Occupying a place amongst these figures we find the Mathematical Man. To some he is a figure of fun, an absent-minded professor eating lilies of the valley in mistake for asparagus. To others he is aloof and sacred, the guardian of eternal truth (for there are many who seem to find no statement more profound for contemplation than $2 + 2 = 4$). George Bernard Shaw, in a passage in which he accuses many branches of knowledge of degeneracy and error, writes, "the tower of the mathematician stands unshaken."

It is clear to those of us who are in some measure mathematicians that the truth lies between these extremes. In order to investigate this more fully we are attempting a survey, details of which are given on a later page. It is hoped to replace the legendary figure of the Mathematical Man by a fully statistical entity attending 7·1 lectures a week and having a predilection for Bach 69·3 per cent. above average.

* * *

It is regretted that the rising costs of printing and paper have forced us to increase the price of EUREKA from 1s. 6d. to 2s. 0d. Every effort will be made to keep the price stable at this figure. It would be particularly appreciated if ex-Archimedeans now working in Universities and Research Labs. throughout the country would undertake to sell copies of EUREKA to their colleagues. Copies may be obtained on a sale-or-return basis from the Business Manager, and a small commission is paid for sales over half a dozen.

The Archimedeans

AT our evening meetings this year we have heard very interesting and lively lectures. In the Michaelmas Term Professor Frisch described some knotty problems concerned with tablecloths and hose piping; Professor Newman measured curves; Professor Semple drew maps of the mathematical world and Professor McVittie spoke on the universe as a whole. In the Lent Term Professor Turnbull spoke on Nothing, Professor Besicovitch on convex figures and Mr. Langford told us how mathematics should be taught. Dr. Todd very kindly showed us the mathematical models in the Arts

School and the four tea-time meetings were addressed by Messrs. Drazin, Williamson, Bullen and Toulmin. Some members visited the Observatory, and at the Annual Problems Drive members donned their thinking caps.

Throughout the year the Music Group and the Bridge Group have held weekly meetings.

The main social events, namely the Christmas Party and the Dance, were very successful and enjoyed by all. Other social events included a cycle run, an amble, and a visit to the theatre.

Once again we have had a record membership and the meetings were well attended.

Finally may I say thank you to the committee and all those who have helped to make the above events possible. I wish the new president and his committee every success.

L. J. M. B.

Professor A. S. Besicovitch

IF, at some time during the last war, you had been wandering through the Great Court of Trinity on a summer afternoon, you might have been privileged to see a distinguished-looking man mowing the lawns in his shirt sleeves. That would have been Abram Samoilovitch Besicovitch, F.R.S.

To the world at large Besicovitch passes for a Russian, but in actual fact he comes from a small and little-known people called the Karaites, a Jewish sect of Turkish origin who are to be found in scattered communities as far apart as Egypt in the south and Finland in the north. Most of them, however, inhabit the area round the Black Sea, and it was there that Besicovitch spent the early years of his life. He himself tells the story of how as a young boy he used to go down to the docks at a small town on the Sea of Azov and ask the English sailors to help him read his English books. On one such occasion he attracted the attention of the captain himself and spent the rest of the time on board as the captain's guest, with a corresponding improvement in the standard of English.

Along with one of his brothers he read mathematics at the University of St. Petersburg, and it was there that he started his academic career, working under the famous mathematician A. A. Markoff on Probability Theory and lecturing, some may be surprised to hear, in Applied Mathematics. With the advent of the revolution life became increasingly difficult in St. Petersburg, and in 1917 he moved to the new University of Perm, in the Urals, where conditions were better but still not really conducive to mathematical research. The rigours of the climate necessitated sitting in a large sack as the only means of keeping warm, and the isolation from the mathematical world was an even more serious obstacle as most of the books in the University were of 1850 vintage and periodicals were an extreme rarity. Despite all these difficulties Besicovitch, during this period, did some work on Real Functions and solved the famous problem of Kakeye. In non-technical language his solution might be described by saying that in order to reverse your car (assumed infinitely thin) you require no room at all, though unfortunately you will have to go off to infinity in an infinite number of directions.

In 1920 he returned to his old University and stayed there until, five years later, he finally left Russia on a Rockefeller research grant and went to Copenhagen. Oxford then claimed him for a year, but fortunately he found his way to Cambridge shortly after, and has remained here ever since.

An analyst throughout his life, he has worked on such topics as Sets of Points, Measure, Real Functions and Parametric Surfaces, not forgetting occasional incursions into other fields such as Number Theory. Perhaps typical of the sort of work he has done is the pathological surface he produced which showed that the definition of area current at the time was completely inadequate. It was the diagram of this surface with its striking similarity to a system of pipes which gave rise to the rumour that he started life as a plumber. Another of his results (though here he was anticipated by Brouwer) was to show that there was no proper equivalent of the four-colour problem in three dimensions, for, even with the obvious restriction to convex polyhedra, the number of colours required turned out to be infinite. With these examples in mind it is not difficult to see why the numbers 0 and ∞ were once described as typical Besicovitch numbers.

Original in other fields as well as mathematics, his leisure hours are spent, not in the orthodox academic pursuit of mountain peaks, but in the quieter joys of long-distance swimming, and the Channel rather than Everest is his goal. Only last year he caused a minor sensation at the Canadian Mathematical Seminar by swimming a mile between lectures. It was there also that he produced his famous card game which has rules so simple that it is played by Russian peasants, but yet resembles Chess in the skill and subtlety of its play. He offered two dollars to anyone who could defeat him, and though some of the younger mathematicians tried hard for a whole month, he returned unbeaten, to the great satisfaction, no doubt, of the Chancellor of the Exchequer.

Since his election to the Rouse Ball chair in 1950 he has, of course, given up supervising undergraduates, and those who were fortunate enough to work for him in earlier days will realise what a loss this is to succeeding years of Trinity freshmen. His supervisions were definitely an experience in themselves; one climbed his stairs in a state of trepidation, contemplating sadly the fate that awaited one's efforts on last week's paper, and wondering what deceptive little problems he would produce this week, problems that charmed by their simplicity yet obstinately refused to be solved. But the rigours of the mathematical instruction were always mitigated by his essential kindness. On one occasion, in order to cheer the despondent pupil, he went so far as to confess that he had never really understood the whole question of pole and polar in elementary geometry.

It certainly cannot be said of many professors that they take such a friendly interest in the undergraduate world as Besicovitch. One of his favourite past-times, he says, is going for walks with undergraduates; indeed so keen is he on these walks that he once assured his young companions that the portending blizzard was, in Russian eyes, a sign of mild weather and need not deter them from their walk. On the occasion of the Commemoration dinner with the subsequent intermingling of High and Low Tables, he is always to be seen in the centre of an animated group, relating some of his popular anecdotes and signing menus for the young autograph hunters. In fact, despite all the minor eccentricities and characteristics which are almost *de rigueur* in a professor of mathematics, the most lasting impressions one carries away are of his extreme amiability and his keen sense of humour. M. F. A.

Supersonic Airflow

By Professor M. J. Lighthill,

Department of Mathematics, University of Manchester

The essential feature, in which supersonic airflow differs from flow at speeds small compared with that of sound, is in the imperfect transmission of pressure changes. Because they are transmitted (except when they are very large) at a speed less than that of the stream, the region influenced by a given pressure change is roughly conical and is mostly downstream of it. In consequence all flows dominated by pressure (broadly, those produced by flow over non-parallel boundaries) are quite different at supersonic speeds, although parallel shearing flows are not qualitatively changed.

One important result is that an expansion of the cross-sectional area of a stream causes a reduction in density and pressure and hence (to conserve energy) an increase in velocity. This is because no relief of the local rarefaction by upstream propagation is possible.

Thus when a boundary wall turns away from a stream it sends out diagonally waves of reduced pressure and increased fluid speed. Each in turn is set more obliquely to the stream than the last because conditions on it are "more supersonic."

When on the other hand a boundary wall turns towards the stream, the waves will tend to set themselves each less obliquely to the stream than the one before it. A little thought shows that this becomes a geometrical impossibility at a certain distance from the wall (or actually at the wall for a sharp bend). The waves then pile up on one another to form a discontinuous "shock wave," in which the fluid motion is throttled, part of the kinetic energy being lost in favour of a pressure increase, and that not with perfect efficiency.

In consequence the normal pattern for supersonic flow past a solid obstacle is a bow shock wave in front, followed by a fan of expansion waves as the stream turns round the obstacle, followed in turn by a rear shock wave as the stream regains its original direction. All these waves stretch diagonally downstream and away from the obstacle, and can be seen clearly on photographs. Their strengths slowly decrease with distance from the obstacle, and their directions therefore all approximate more and more closely to that for infinitesimal disturbances. Thus the bow shock is convex to the oncoming flow, its speed of propagation relative to the parallel flow ahead of it decreasing with distance. Indeed for blunt-nosed obstacles it is actually normal to the stream (and detached) just ahead of the obstacle, leading to a region of subsonic flow around a nose stagnation point. The rear shock, however, is concave, because it is the flow *behind* it which is nearly parallel, and its speed relative to this increases with distance.

When the obstacle is placed in the working section of a supersonic wind-tunnel, the shock waves described above are reflected in the walls. However, the reflection does not satisfy Snell's law exactly because the shock waves are not weak enough to behave exactly like sound waves. If the reflected shock waves pass behind the obstacle, then the flow past the model is unaffected by the presence of the walls; i.e., there is no "wind tunnel interference." With a large model, when the bow shock is still very strong even at the wall, ordinary reflection may become impossible, because the air cannot take both the compressions at the shock waves while remaining supersonic. The regime is replaced by an irregular kind of reflection involving a nearly normal shock wave and a region of subsonic flow; and for larger obstacles still even this regime may break down and a normal shock wave appear right across the tunnel.

As was foreshadowed earlier, the "boundary layer" attached to any solid surface, of heavily sheared fluid across which the pressure is constant, behaves less differently at supersonic speeds: that is to say, its reaction to a given distribution of velocity in the main stream along the surface outside the boundary layer is not greatly altered. For example, it will become turbulent for a Reynolds number (velocity times length of layer, times density, divided by viscosity) of order 10^5, with an increased tendency to turbulence in flow which is decelerating and a decreased tendency when the conditions of the oncoming stream and the solid surface are particularly smooth. The boundary layer will thicken in retarded flow, and separate if the retardation is too great; this is because the fluid with low kinetic energy near the wall is too greatly slowed down by the higher pressure ahead of it. A turbulent boundary layer thickens less rapidly than a laminar one under retardation of the main stream, and separates much less readily, because the fluid near the wall is constantly being renewed with fluid of higher kinetic energy.

However, there are now some temperature effects. If fluid at supersonic speeds is reduced to rest adiabatically, its kinetic energy goes directly into heat (the temperature increasing by anything from twenty per cent. upwards). In the boundary layer the process is far from adiabatic, since energy is dissipated by friction, and also conducted into the solid surface and into the main stream. However, under conditions when the solid takes a negligible amount of heat, because it is a poor conductor, or because it has already warmed up to the temperature of the adjacent air, and is not radiating very rapidly, one finds that the remaining effects (additional heat generated by friction, and conduction into the stream) approximately cancel out, so that the heat energy at the surface is nearly equal to the total energy of the stream.

The fluid near the surface is therefore lighter; it is also more viscous (gases, unlike liquids, become more viscous with temperature) and so its Reynolds number is reduced. Hence transition to turbulence is somewhat less likely than at low speeds. Evidently, too, it is more easily slowed down by pressure gradient, and so thickens and separates a bit more readily.

Passing to some phenomena which are only now in the process of elucidation, it should be noticed that although the effect of pressure gradient on boundary layer behaviour is only slightly altered at supersonic speeds, the effect on the pressure gradient in the irrotational flow (outside the boundary layer) of the *shape* of surfaces bounding it, including the shape of the boundary layer itself, is greatly changed as indicated at the beginning of this article. This makes possible new kinds of reciprocal interaction between boundary layer shape and main stream pressure gradient, which cause any local disturbance at the surface to have an upstream influence ("propagated up the boundary layer," as it is sometimes, loosely, described), contradicting the rule that this is impossible in supersonic flow.

There are two main kinds of interaction resulting in such upstream influence:

(i) thickening, resulting in increased pressure, resulting in more thickening (or thinning, resulting in decreased pressure, resulting in more thinning);
(ii) separation, resulting in a pressure rise just ahead of the separation point, resulting in separation there too, and so on until the pressure rise is made gradual enough for further separation to be avoided.*

The second mechanism operates when the disturbance is compressive (e.g., when a shock wave is incident on the layer, or the wall bends in towards the stream), provided it is strong enough to cause separation (which is almost always the case for laminar layers, but not necessarily for turbulent ones). It is associated, when the layer is laminar, with large distances of upstream influence, increasing with the strength of the disturbance but decreasing with Reynolds number (because at high Reynolds numbers the separated layer becomes turbulent sooner). A spectacular example is flow past a bluff body with a thin probe sticking out in front. The boundary layer on the probe separates, forming a cone of dead air which fairs off the body and may halve its drag.

The first mechanism, associated with much smaller lengths of upstream influence, may also operate under these conditions, but can be neglected except in flows where these conditions are absent. For example, when the wall turns away from the stream the pressure begins to fall, while the boundary layer thins, some six to ten layer thicknesses ahead of the corner.

The boundary layer also separates at the base of a flat-based body and turns, not through the full angle of 90°, but through an angle of nearer 12°, thus focussing itself before becoming the turbulent wake. The angle turned through, which controls the suction at the base, is influenced by the amount of air which has to be fed into the dead air region (behind the base) at the point of origin of the wake, in order to supply air to be in turn entrained into the separated boundary layer. The angle, and hence also the base suction, can be reduced by introducing additional air into such a dead air region through holes in the base.

* Unlike mechanism (i), this or something like it can also operate at low speeds.

The qualitative considerations discussed in this article can all be given quantitative value by mathematical calculations. These have resulted in a substantial collection of data on supersonic flow which has progressed in association with experiments. Now the work (as well as being continued) is being extended into the transonic region (where the coexistence of subsonic and supersonic flow regions is a complicating factor) and the hypersonic region (where new considerations from high temperature and low density physics may be needed).

Mathematician's Lament

(*to be sung to the tune of "Clementine"*)

Three long years of mathematics
Are too much for any man.
Here before you, I implore you,
Why not leave it while you can?

What will happen to a sequence
As n nears infinity?
All these queries about series
Are the limit, you'll agree.

I think div. x is as complex
As the curl—indeed I find
I have *no* bent for the gradient
For I'm not that way inclined.

Mr. Hardy's complex theories
Much discussion will incite;
There are no laurels in these quarrels
Because Hardy isn't Wright.

If you all knew what's before you
When you first come here with pride;
There'd be fewer doing Pure,
Even less would do Applied!

A. B.

Fancy That!

A certain distinguished applied mathematician was heard to assert in his lectures: "And so the problem can now be solved without any mathematics at all—just by the use of group theory!"

Cosmological Theories

By F. PIRANI

THE application of scientific method to the study of physical systems depends on the possibility of deciding which of the quantities involved shall remain fixed during a series of observations, and which shall be allowed to vary. Further, the possibility of examining a number of similar systems enables the abstraction of their common characteristics to form physical "laws" which apply to all the systems independently of the particular conditions which distinguish them one from another. In cosmology, neither of these possibilities exists. Thus one is unable by experiment or observation to decide which of the characteristics of the universe represent the working of a physical "law" and which of them are accidental. (When only one system can be observed, this distinction loses its meaning.)

As a consequence, the cosmologist appears to have much greater latitude in the construction of mathematical descriptions of the system he observes than do other physicists. There are two distinct approaches to the construction of cosmological theories, which we shall call inductive and deductive. The inductive cosmologist seeks to describe the characteristics of the universe by suitable modification and extrapolation of local physical laws. The deductive cosmologist makes general hypotheses about the universe and attempts to deduce from these both the local laws and the observed large-scale characteristics. Most theories are constructed by a combination of these two approaches, but it is seldom clear which parts of a given theory are constructed by which approach.

There is strong evidence that the universe is homogeneous and isotropic when viewed on a sufficiently large scale, but there is little evidence about its evolution during times of order greater than 10^9 years. Theories which agreed with each other and with observation about the present state of the universe might differ widely concerning the state in the distant past and in the distant future, and there is no observational criterion by which to prefer one such theory to another. It is the view of many cosmologists, however, that an additional criterion is provided by the uniqueness of the universe. A satisfactory theory should lead unambiguously to a unique mathematical description which agrees with the observations and which is deduced without the arbitrary rejection of alternative descriptions consistent with the original hypotheses but inconsistent with observation. To put it in another way: if a physical system is such that n of the quantities involved can be assigned arbitrarily, then the theory describing systems of this type may be expected to contain n arbitrary parameters. Hence a cosmological theory may be expected to contain no arbitrary parameters at all.

We shall consider some of the more important of the general hypotheses on which cosmological theories have been based, namely the principles of Copernicus, Bondi and Gold, Poincaré, Einstein, Mach and Eddington.

The principle of Copernicus, often called the narrow or restricted cosmological principle, asserts that, viewed on a sufficiently large scale, the universe presents the same overall aspect to an observer at rest anywhere in it (the meaning of "sufficiently large" we shall examine below; the definition of "at rest" presents no serious difficulties, but requires considerable discussion). The mathematical description of such a universe must clearly be covariant under space-translations of the origin.

The principle of Bondi and Gold, called the wide or perfect cosmological principle, asserts that the universe presents the same overall aspect not merely to observers at rest in different places, but also to observers making their observations at different times. In this case the mathematical description must be covariant under translations in time as well as in space.

The principle of Poincaré, usually known as the principle of special relativity, and frequently attributed to Einstein or to Lorentz, requires that local physical laws take the same form for observers in states of uniform motion with respect to one another. The descriptive equations must be covariant under a certain continuous group of transformations, namely the Lorentz group.

Einstein's principle of equivalence asserts that the totality of local observations possible to an observer do not enable him to distinguish whether he is in a gravitational field or undergoing an acceleration. The equality of gravitational and inertial mass is an immediate consequence of this hypothesis. A mathematical formalism in terms of which the principle may conveniently be expressed is one covariant under the group of all continuous differentiable non-singular co-ordinate transformations.

Mach's principle of inertia can be formulated in a variety of ways. In a general form, it asserts that kinematically equivalent motions are dynamically equivalent. For example, an observer cannot, according to this principle, distinguish between the state in which he is at rest and the remainder of the universe rotates about him, and the state in which he rotates and the remainder of the universe is at rest. In another form, the principle asserts that the inertia of a body is determined by all the other matter in the universe. "There can be no inertia of matter against space," says Einstein, "but only inertia of matter against matter." Few mathematical formulations of Mach's principle have been attempted, and so far none has proved completely satisfactory.

Eddington's principle, which is widely misunderstood, has been clearly expressed by Whittaker. "All the quantitative propositions of physics, that is, the exact values of the pure numbers that are constants of science, may be deduced by logical reasoning from qualitative assertions, without making any use of quantitative data derived from observation." This is quite different from the epistemological extremism often attributed to Eddington. The mathematical formulation of the principle by its author is based on a generalised tensor calculus whose number system is constructed from the direct square of quaternion algebra.

Besides the principles, there are a number of theoretical concepts common to cosmological theories. Central among these is the idea of the *substratum*, the ideal smoothed-out background which ignores the irregularities of the actual universe, such as the concentration of matter into nebulae, and preserves only those large-scale characteristics, such as the mean overall density and velocity of matter, which are supposed to be unaffected by the smoothing-out. To see the significance of the substratum, let us consider the narrow cosmological principle. This principle does not agree with observation if the scale of view of two observers under consideration is measured by a volume of as little as 10^{12} cubic light-years, because some such volumes contain nebulae and others do not. However, if the substratum were the real universe, then the narrow cosmological principle would be satisfied for observers with any scale of view.

The misapprehension that all cosmological theories are based on Einstein's theory of general relativity may be attributed to an historical circumstance. The subject was practically dormant when Einstein revived it with his famous paper of 1917. The stimulus which he supplied has produced an immense variety of theories in the intervening 35 years. We shall examine some of the better-known ones in relation to the general principles already described.

Milne's theory of kinematical relativity, which is largely deductive in character, is based on detailed application of the principle of Copernicus to a continuous system of observers, each of whom measures time by a clock which has been compared with the clocks of other observers by means of light signals. Possible motions of the system of observers, which constitutes the substratum, are deduced axiomatically. The principal difficulty confronting this theory, whose whole development has been confused by controversy, appears to be its inflexibility. The substratum is dealt with satisfactorily but difficulties arise as soon as the actual universe, taking account of the discrete nature of the nebulae in contrast to the continuous nature of the substratum, is considered.

The steady-state theories of Bondi and Gold, Hoyle and McCrea, incorporate the wide cosmological principle. In the theory of Bondi and Gold, this principle forms a basis on which the theory is built. The theories of Hoyle and McCrea are based on a modification and a re-interpretation respectively of the field equations of general relativity, and consequently suffer from some of the defects of relativistic cosmologies. The theory of Bondi and Gold satisfies the criterion of uniqueness, but has no field-theoretical form. In all forms of the steady-state theory it is the case that matter is being created throughout the universe, but at a rate too small to be observed by known methods. The relation to Mach's principle is not yet clear for any form of the theory.

Relativistic cosmology is based on solutions of the field equations of general relativity which satisfy the principle of Copernicus. General relativity in turn is based on the principle of equivalence. Einstein was strongly influenced by Mach, but nevertheless was unable to incorporate Mach's principle into the theory, and later workers have had little more success in this respect. The criterion of uniqueness is not satisfied, and there is disagreement with observation for nearly all relativistic cosmologies.

Eddington spent the last twenty-five years of his life constructing a physical theory based on his principle. The theory, like the principle, is widely misunderstood. The criterion of uniqueness is in some respects satisfied, and in the early forms of the theory Mach's principle appears to be satisfied also.

As can be seen, no completely satisfactory cosmological theory exists at present. Theories of all degrees of complexity and of all degrees of disagreement with observation can be constructed by selecting a formalism and some principles and fitting them together. For most cosmologists, the choice of a preferred theory will be an aesthetic one, since the observational evidence does not at present provide a sufficiently clear criterion.

References.

Bondi. *Cosmology.* Cambridge, 1952.
Eddington. *Fundamental Theory.* Cambridge, 1946.
Einstein. *The Meaning of Relativity.* 3rd Edition, Princeton, 1950.
Milne. *Kinematical Relativity.* Oxford, 1948.

Cinderquadrics

(or the Babes in the Complex)

ONCE upon a time there was a beautiful vector space called S_3. In this space there lived a father quadric called S, a mother quadric called S′, and a sweet little twisted cubic called Γ′, who appeared when mummy and daddy quadric had a generator in common. (They had wanted to call her Γ, but being a girl she was not a male point cubic but its osculating developable.)

Γ′ was very popular amongst her friends. They all envied her beautiful parametric form (θ^3, θ^2, θ, 1) and tried to copy her by having their tetrahedra of reference altered. Unfortunately, however, they did not know the transformation.

When Γ′ grew up she was wooed by a quadric T. When S came to hear of this he was very angry, for T was nasty and degenerate. Even in his youth he was so bad that the determinant of his form vanished and now he was so second rank that he was only a pair of planes. S was firm, and when he threatened to reciprocate T into a pair of points, T left Γ′.

She was heart-broken by her cruel father and ran away into a linear complex to hide. She could see that it was linear and found it very complex because nobody had told her about S_5; consequently she got lost.

Suddenly an equation—

$$a\,x\,x^* + b\,x + c\,x^* + d = 0$$

appeared and said, "I am your fairy godmother. I am a (1–1) correspondence and evidently algebraic." "Oh !" said Γ′, "Does everyone have a fairy godmother ?" "Well there is a quartic curve," replied the fairy, "who hasn't got one, but speaking confidentially he's not quite rational." "Can you get me out of this horrible complex, fairy godmother?" asked Γ′. Before you could simultaneously diagonalise two quadratic forms the fairy godmother muttered the magic words "C. V. Durell" and ω and ω' became the circular points at infinity.

"Where am I ?" murmered Γ′. "In Euclidean space," replied the fairy. "What's this funny stuff ?" "Distance. You can add it and subtract it and it's a good thing. Use it wisely and you will find your way home. But beware ! don't ever mention the magic words to your father. He will consider this solution to your problem unethical."

Γ′ found her way home and found her distracted suitor waiting for her. He was a very handsome pair of planes, $xt = 0$ and she found him irresistible. Indeed, she fell into his arms and he kissed her where $\theta = 0$ and $\theta = \infty$.

S and S′ were so relieved to see Γ′ back home that they forgave her and her suitor. They were soon married and lived happily ever after in that vector space called S_3. H. M.

Relative Values

In attempting to derive some geodesical equations,
Einstein, Infeld and B. Hoffman found unfortunate relations
That exist between the terms of the approximation serial
To Ricci's coefficients (in the absence of material).

The only way discovered to avoid this fact annoying
Was the transient creating and the subsequent destroying
Of a concept most original, the *Dipole Gravitational;*
Though technically necessary, 'twas scarcely observational!

The problem disconcerting which immediately arose
Was to find those masses *negative* which *Dipoles* must compose;
'Twas a question no more physical, but rather philosophical
Which altered modern thinking in a manner catastrophical.

If of this basic problem you are not as yet too weary,
I refer you to the work of Born on *Unified Field Theory*
(Of course, by well-known theorems, all such devious hanky-panky
Is essential for obeying the identities Bianchi).

Julius.

Definition

Geometer: One who studies geometry; a caterpillar.

Shorter Oxford Dictionary.

Lebesgue Measure
Is a vague pleasure,
But the sets of Borel
Are absolute hell.

P. A. S.

Three Dimensional Cross Number Puzzle

By NERO

a	*b*	*c*
d	*e*	*f*
g	*h*	*i*

j	*k*	*l*
m	*n*	*p*
q	*r*	*s*

t	*u*	
	v	
w	*x*	*y*

The blocks are successive vertical sections of a cube of which every vertical and horizontal section contains each digit from 1 to 9 once only. In the clues "a" represents the digit occupying the square marked "a," etc.

Clues:—

abc = amw
ceg = wqg
cfi = wri
env = qnl
hnu = jns
beh = ihg − fed
ghi = 2 (ihg − aei)
beh = jkl − xvu
aei = jkl − yvt
spl = wxy − yvt

Solution to Cross Number Puzzle in Eureka No. 14

A = 23	B = 7	C = 21	D = 4	E = 18	F = 2	G = 16
H = 30	I = 13	J = 27	K = 10	L = 24	M = 8	N = 22
P = 5	Q = 19	R = 3	S = 17	T = 31	U = 6	V = 11
W = 14	X = 20	Y = 25	Z = 28.			

The Mathematical Association

President: K. S. SNELL, ESQ., M.A.

The Mathematical Association, which was founded in 1871 as the *Association for the Improvement of Geometrical Teaching,* aims not only at the promotion of its original object, but at bringing within its purview all branches of elementary mathematics.

The subscription to the Association is 21s. per annum: to encourage students, and those who have recently completed their training, the rules of the Association provide for junior membership for a limited period at an annual subscription of 10s. 6d. Full particulars can be had from The Mathematical Association, Gordon House, Gordon Square, London, W.C.1.

The *Mathematical Gazette* is the journal of the Association. It is published four times a year and deals with mathematical topics of general interest.

Differentiation

The first issue of Eureka contained a questionnaire designed to investigate the habits and eccentricities of mathematicians. Thirteen years have elapsed since then and we think it may be of interest to repeat the experiment. The questionnaire given below is based upon the original, though changes have been made where necessary. It is hoped that as many of our readers as possible will complete it and either give it to one of the Editorial Committee or send it to:

J. C. Polkinghorne,
Trinity College,
Cambridge.

A report on the findings will be published in the next issue of Eureka.

Age: Sex: Year at University:

Is there any evidence or tradition of mathematical ability in your family? State what sort, if any.

Do you: (*a*) frequent concerts?
(*b*) make, or attempt to make, any sort of musical noise?

Place in order of preference: Bach, Beethoven, Chopin, Debussy, Mozart, Wagner.

Do you read poetry? If so, whose?

How often do you read "Alice in Wonderland"?

Do you: (*a*) read; (*b*) compose; nonsense rhymes?

What do you think of Gilbert and Sullivan?

Do you play bridge, chess, crosswords, noughts and crosses?

Do you knit, sew, embroider, darn your own socks?

Does your hair come out in large quantities during term?

When working do you:

(*a*) smoke? (*b*) eat sweets?

(*c*) drink? (*d*) pace up and down?

(*e*) roll on the floor *in extremis*?

State your other little habits.

Insert specimen of lecture note writing.

Do you consider yourself a mathematician, a student of mathematics, or do you merely "read maths"?

Which branch of mathematics do you prefer?

How many hours a week do you work in term?

Name your favourite lecturer.

What is your opinion of the value of:

(*a*) lectures?

(*b*) supervisions?

Given the choice would you still read mathematics here?

If not, what?

What is (*a*) your religion? (*b*) your politics?

What are your principal relaxations?

What do you think of this questionnaire?

Book Reviews

An Introduction to the Theory of Control in Mechanical Engineering. By R. H. MacMillan. (Cambridge University Press.) 30s.

This book must not be regarded as a treatise, but rather as an elementary text book on the subject considered (although its binding does not recommend it as such). As the book is intended for engineers the author develops the theory of control by studying numerous special cases and examples. He does not give any account of the general theory until late in the book and the reviewer feels that this makes it rather difficult to follow the line of argument. The mathematical techniques used are all of an elementary character so that the problems considered are somewhat restricted; no study of the mathematical theory of non-linear systems is made.

There is no reference in the extensive bibliography to the work of Wiener or to the theory of information which might be considered relevant. A mathematician would find the notation used and the method of approach to problems rather difficult. There are numerous clear diagrams showing the apparatus required to effect control in various types of mechanical system. C. B. H.

Operational Calculus based on the Two-sided Laplace Integral. By Balth van der Pol and H. Bremmer. (Cambridge University Press.) 55s.

This is an important book. It is based on the relations

$$f(p) = p\int_{-\infty}^{\infty} e^{-pt}\, h(t)dt \qquad \alpha < \operatorname{Re} p < \beta$$

$$h(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{pt}\frac{f(p)}{p}\, dp \qquad \alpha < c < \beta$$

and gives a multitude of results, some useful, some elegant, and some curious, based on them. The emphasis is on the mathematical method, though many physical applications are given. The main theme is to work up to the solution of linear ordinary and partial differential equations; there are chapters on the delta function, asymptotic approximations, difference equations, and integral equations, and the book ends with a list of transforms. The power of the method in proving identities is well illustrated; there are even applications to the theory of numbers.

In comparison with other works on the Laplace transform the novel feature is that the lower limit in the first integral is taken as $-\infty$ instead of 0, with the consequence that in general if the integral converges at all it does so in a strip instead of a half-plane. The usual case is of course included by replacing $h(t)$ by $h(t)\,U(t)$, where $U(t) =$ 0 for negative t and $=1$ for positive t.

Most of the treatment is based on the first of the above equations, and other works are referred to for many of the problems that arise when $h(t)$ has to be found from $f(p)$. The two-sided transform gives extra generality, but the free manipulation of the contours is sacrificed. Watson's lemma and the method of steepest descents are not used in the sections on asymptotic approximations.

It is a welcome sign that the authors, like McLachlan, restore the multiplier p in the definition of $f(p)$, so that even when t is dimensional $f(p)$ and $h(t)$ have the same dimensions. But it is a pity that they did not call their book simply "The two-sided Laplace transform." After a few paragraphs on Heaviside's achievements, in which the definition of p^{-1} as the operation of definite integration from 0 to t is mentioned, the authors switch to a definition of p as an ordinary complex variable. They say that they have not found in Heaviside's work general rigorous statements on the question, but do not mention other places where such statements exist. They show no awareness that the p^{-1} operator provides a standard method of proving the existence of solutions of differential equations, and works for such an equation as

$$dx/dt + \alpha x = \exp(a^2t^2)$$

for which the Laplace transform breaks down at the outset.

The one-sided and two-sided Laplace transforms are in fact important parts of complex variable theory, and the authors have done a great work in showing their power. But they are neither equivalent to the operational method nor adequate substitutes for it. H. J.

Elements of the Topology of Plane Sets. By M. H. A. NEWMAN. (Cambridge University Press.) 27s. 6d.

The theory of functions, particularly functions of a complex variable, makes great use of topological ideas and topological theorems. One of the principles which guided the selection of material for this book was that of trying to provide rigorous proofs for some of the basic theorems; among these are Jacobi's theorems about implicit functions, Jordan's theorem about a closed curve in the plane and Cauchy's theorem on complex integration in its strongest form, all of which are proved here.

There are two halves to the book, although in the second edition no recognition of this is made. The first half is occupied with Analytic Topology of a general metric space. Although these chapters are perhaps primarily introductory, they contain some results included for their intrinsic interest—for instance the proof that every locally connected continuum is the continuous image of a segment—this half closes with the topological characterisation of the segment and the circle.

In the second half combinatorial methods are introduced. The chief tool is that of chains on a rectangular grating on the plane; for coefficients integers mod 2 are used. With this not very elaborate apparatus simple and beautiful proofs are given of Alexander's Lemma and Jordan's Theorem. There is a chapter on domains in the plane, the relation of a domain to its frontier and the comparatively unfamiliar converse to Jordan's Theorem. The last chapter concerns homotopy properties, intersections of arcs and the order of a point with respect to a closed curve.

The changes introduced in the second edition do nothing to affect the general character of the book; it remains readable, attractive in appearance, clear and in general excellent for the purposes for which it was written. The new features consist of an enrichment of the text at various places (for instance by discussion of normed vector spaces, peano continua and Jacobi's Theorem, in suitable contexts), the omission of sections on boundary elements of domains and connectivities of closed sets, and the addition of the section on intersections of curves. There are a few examples and exercises scattered through the book.

The book as a whole is a fine advertisement for Topology; it displays its elegance, its rigour and its relevance to analysis. It would serve well as an elementary introduction to Analytic Topology; but, although combinatorial methods are used, they occur in too special a form to make a good introduction to homology theory. S. W.

Conformal Representation (Cambridge Tract No. 28, 2nd Edition). By G. CARATHÉODORY. (Cambridge University Press.) 12s. 6d.

The first two chapters of Prof. Carathéodory's Tract provide a valuable approach to conformal representation and the concept of the non-Euclidean geometry of the circle developed there occurs constantly in the rest of the Tract. It gives a geometrical insight into a number of proofs of theorems which would be lacking in purely formal methods.

In the new chapter VIII which has been added in this second edition the problem of the conformal mapping of surfaces is developed further. In chapter VII it is shown that if a surface is topologically equivalent to a sphere and each point of it lies in a portion of the surface which can be mapped conformally on a portion of the plane, then the whole surface can be mapped conformally on the sphere. In chapter VIII a general multiply-connected Riemann surface is considered as built up from a framework of triangles T_j each of which has exactly one side associated with one side of another triangle. A surface is said to be closed if there are a finite number of triangles, open if there are an infinite number. A fixed vortex O is chosen and each triangle T_j associated with a set of triangles $\bar{T}_j^{(r)}$, $r = 1, 2, \cdots$, the triangles $\bar{T}_j^{(r)}$ being in one-one correspondence with the homotopically distinct paths from O to T_j. The sides of the $\bar{T}$ triangles are then associated in an obvious way so as to form a simply-connected surface $\bar{S}$, known as the universal covering surface of S.

If S is already closed and simply-connected the result of chapter VII applies (and $S \equiv \bar{S}$). Otherwise $\bar{S}$ is open and it is shown that $\bar{S}$ can then be mapped conformally on either the Euclidean plane or the interior of a circle $|t| < R$. Since the metric in these two cases is fundamentally different an examination of S will sometimes determine easily which is the appropriate mapping. F. R. K.

The following books have been received and reviews will appear in our next issue.

Lebesgue Integral. By J. C. BURKILL. (Cambridge University Press.)

Methods of Algebraic Geometry. (Vol. 2). By W. V. D. HODGE and D. PEDOE. (Cambridge University Press.)

Introduction to Modern Prime Number Theory. By T. ESTERMANN. (Cambridge University Press.)

Cosmology. By H. BONDI. (Cambridge University Press.)

Inequalities (2nd Edition). By G. H. HARDY, J. E. LITTLEWOOD, and G. PÓLYA. (Cambridge University Press.)

Mathematics by the Fireside. By G. L. S. SHACKLE. (Cambridge University Press.)

Mathematics, the Queen and Servant of Science. By E. T. BELL. (Bell.)

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Copies of EUREKA Nos. 10 and 11 (2s. each, post free) and Nos. 9, 12, 13 and 14 (1s. 6d. each, post free) are still available. Cheques, postal orders, etc. should be made payable to "The Business Manager, Eureka."

The Editor still requires copies of Nos. 1 to 7 and would be glad to hear from any reader willing to sell these. Photo-copies of these numbers may be obtained from the Science Museum Library, South Kensington, London, S.W.7, or through the Philosophical Library, The Arts School, Bene't Street, Cambridge.
