Eureka Editor
The Archimedeans
Centre for Mathematical Sciences
Wilberforce Road
Cambridge CB3 0WA
United Kingdom

archim-eureka@srcf.net

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Editor: *O. M. Phillips (Trinity)*
Business Manager: *J. L. Martin (King's)*

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Contributions and other communications should be addressed to:

The Editor, "Eureka,"
Arts School,
Bene't Street,
Cambridge,
England.

The Archimedeans

In spite of a rather large financial loss instead of the usual profit the Society can, I think, claim to have had a most successful year. The membership continues to rise although there were fewer new members than in recent years.

Attendance at our evening meetings was, on the whole, good and on two occasions the house was full—for Professor Aitken's dazzling display of high speed arithmetic, and for Sir William Penney's "Explosions." In the Michaelmas term we also heard Professor Whittaker on "Representation in Series" and Mr. Hope-Jones on "Fun with Probability." In the Lent term Dr. Turing talked about fircones, Mr. Tregenza on how to teach mathematics at school and Professor Broadbent entertained us with "A Second Empire Comedy" with a moral for those who would combine nationalism with mathematics. I should like to record our gratitude to all these speakers, most of whom came to Cambridge solely to give their lectures, usually having to return home early the following day.

Our tea-time meetings were all suitably light-hearted and included a symposium in which three speakers put forward the merits of Euclidean and non-Euclidean geometries. The speakers at the other meetings were Mr. Atiyah, Mr. Penrose and Mr. Polkinghorne. On the entertainment side, the Music Group and the Bridge Group continue to flourish; in the Easter term punts were provided for the madrigals, and in the Michaelmas term we held a Christmas Party and arranged a visit to the Gilbert and Sullivan opera at the Arts Theatre. Mention must also be made of the Problems Drive and I think those who attended it will agree that it can be classed with the entertainments.

A new layout for the Society's card has been introduced and although probably not yet settled in its final form, it is obviously a great improvement.

Finally, I should like to thank all who have helped the Society in the last year, both on the committee and off it, and to wish the new President and her committee every success.

P. S.

"What shall we say of the mathematics? Shall we deem them to be the delirious ravings of madmen? Nay; we cannot read the writings of the ancients on these subjects without the highest admiration."

John Calvin, *Institute.*

Are Mathematicians Musical?

SOME time ago, Mr. E. H. Lockwood of Felsted School, Essex, had the opportunity of asking a mixed group of 88 teachers of mathematics to express a preference for one of the arts. At his suggestion, similar figures were obtained in Cambridge.

The survey at Cambridge was made among students reading the Mathematics Tripos and also a number of post-graduate people engaged in research in mathematics, a total of 79. A "preference" was taken to mean either participation or a reasonably active interest in the particular branch of art. The results are given below, and are expressed as percentages of the total numbers interviewed. Mr. Lockwood's figures from Essex are given in the first column, the Cambridge figures in the second.

Music	40	47
Drama	17	6
Architecture	10	5
Prose	8	14
Painting	7	11
Ballet	6	8
Opera	6	5
Poetry	5	2
Sculpture	1	2

These results are quite interesting, although their precise significance cannot be evaluated without further information. The most striking point is the overwhelming preference for music in both cases. This may at first sight support the belief that mathematicians are "musical"; but on the other hand since music, of all the classifications above, is probably the most readily available, this may merely reflect human laziness. Recent demonstrations by Hans Haas have shown that fish, also, prefer music. Clearly, for comparison, a similar sample of non-mathematicians is required.

Some differences between the two groups which may perhaps be significant include the relative lack of appreciation of drama and architecture in the Cambridge group, whose average age is presumably younger than that of Mr. Lockwood's group. Does appreciation of these mature with age?

Hymne to Hymen

(Ce poème, dédié par Blanche Descartes à M. Hector Pétard à l'occasion de son mariage, est actuellement réimprimé avec l'aimable autorisation de ce dernier.)

"Love came in in great excitement, and said,
'My dream has come true.'
He had found a solution to the biharmonic equation" . . .
from a lecture by Prof. D.

The wind was blowing soft, the sun was sending
along their space-time geodesics wending
millions of photons, orange, green, and yellow,
making the scene enchanting, warm, and mellow,
as by diffraction or reflection they're diverted
into the eye, and so to sight converted.
Their energy, or frequency, is reckoned
in million kilocycles to the second.

Love, from his pedestal in Piccadilly
is duly energised, and willy-nilly
a billion arrows sends with high velocity . . .
for he's a chap of great precocity . . .
a billion? . . . more, in the vicinity
of undenumerable infinity.

The photons, $h\nu = E$ obeying,
set many orbital electrons swaying.
The arrows, which ignore Dirac's equations,
cause sinuosidal cardiac palpitations
in every youthful bachelor and spinster
within a neighbourhood of old Westminster.

Hector Pétard, that well-known big-game hunter,
feared that his intellect was growing blunter.
A variation problem had him really nettled,
he couldn't see how it could well be settled:
who was it maximized charm, wit, and grace,
and was the fairest of the human race,
yet minimized, under those same conditions,
all sorrowful and nasty dispositions?

An arrow, flying straight without deflection,
with Hector Pétard's heart made intersection
(as well it might, with probability p,
there's arrows almost everywhere, you see).
Sensing a sudden break in his dejection,
he gave a glance in a north-west direction
(direction cosines θ, θ, 0,
where $\theta^2 = \frac{1}{2}$). Our hero
saw there a lovely maiden, smiling gaily,
reading the *Telegraph*, or some such daily.
Seeing his look, she too felt quite elated,
and so his greetings were reciprocated.

Her conversation gave great satisfaction,
they had a strong Newtonian attraction.
She was kind-hearted, generous, and thrifty,
her I Q very much above 150,
her dazzling figure, when it was in focus,
beat hollow any algebraic locus,
her legs enveloped gracefully in nylon.
She solved his problem to within ϵ.
And when his feelings he had deeply sounded,
he found his love for her was quite unbounded.

"You are my true reciprocal," quoth he, . . .
"And you my contragredient," said she. . . .
"My converse, dual, polar, transposition,
my only isomorph in disposition,
my image, inverse in the sphere of life,
for future time why not become my wife?" . . .
"Your company is very sweet communion:
I think our meet ought to include our union." . . .
"The union is contained within the meet
only of equal sets. The proof is neat.
Come, charming one, let's be identified:
I'll be the bridegroom, you the blushing bride."

Our couple at this moment hail a carriage.
"Hey driver, speed to church. We want a marriage."
The vicar seeing Hector questioned whether
they're certain he and she would live together
connectedly as long as they drew breath,
and never separated but by death?

"Oh, yes, we're positive. Oh, absolutely,
and swear by Harold Jeffreys resolutely;
this is no deviation due to chance,
it has statistical significance."
So while he handed him the golden torus,
the vicar, quickly marrying them before us,
explained, as he performed the operation,
"This is an irreversible transformation.
You, Hector, owe to her in calm or storm,
convergence absolute and uniform,
    by involution you're uniquely mated,
    in fact, harmonically conjugated."

Then Hector turned to her after the mating,
and ceased to oscillate, but osculating,
he said, "Beloved one, now you are mine,
and I am ever yours, this ring's a sign
of an implicit perfect right ideal.
Of us two conjugates, the sum is real,
the difference is pure imaginary,
and ever negligible it shall be.
    We two are definitely integrated,
    and never shall be differentiated."

Like logs, on adding them they multiply.
No conservation law can here apply,
but step by step, proceeding by induction,
a joyful family is in construction.
    Best wishes from their friends in Trinity
    mount steadily towards infinity.

The moral of this episode is sweet.
Their hearts in resonance together beat.
In harmony, and equal in persuasion,
they form a Biharmonical Equation.
    But though this romance has us thrilled, enraptured,
    we're not quite sure the lion's really captured.
    Sometimes we wonder if, when we confront her,
    we'll find the lion's swallowed up the hunter?

BLANCHE DESCARTES.

December 1945.

Problems Drive, 1953-54

1. ABCD is a rectangle, and P is a point such that APBCD are concyclic. The line PQR is parallel to AD, meeting AB in Q and CD in R. The point O is taken in AC such that OP is perpendicular to AC. The line OR meets AD in X, and OQ meets BC in Y. Prove that XPY are collinear.

2. The Boy Scout offered to guard their diminishing store of cigarettes for the night, but they thought it wiser each to take a watch, to watch him. During the night, each in turn divided the cigarettes, gave the boy the remainder to keep him quiet, and took his share. In the morning, they divided the rest equally once more, and gave the boy the remainder. What was the least number they could have started with if there were three of them, and the number the boy got was one each time? (The general solution can be obtained in two steps.)

3. What is the next term in each of the following series?
   (*a*) 0, 1, 3, 6, 10, 15, . . .
   (*b*) 0, 1, 3, 7, 15, 31, . . .
   (*c*) 0, 1, 3, 8, 21, 55, . . .
   (*d*) 4, 6, 9, 10, 14, 15, 21, 22, 25, 26, 33, 34, 35, . . .

4.
$$\begin{array}{r} \text{ONE} \\ \text{TWO} \\ \text{FOUR} \\ \hline \text{SEVEN} \\ \hline \end{array}$$

   S is not zero, and no digit is repeated.

5. Express in terms of four fours and normal arithmetical symbols:
   (*a*) 37, (*b*) 57, (*c*) 77, (*d*) 97, (*e*) 123.
   (Example: 27 can be written $(\sqrt{4})/{\cdot}4 + 4! - \sqrt{4}$
   or $(4/{\cdot}\dot{4})\sqrt{(4/{\cdot}\dot{4})}$.)

6. A double four is a set of four skew lines in four dimensions and their (unique) transversals taken in threes. The opposite of a line is the transversal which it does not meet. The axis of a pair of lines is the join of their meets with each other's opposite.
   Prove that the axis of one pair of lines (or their opposites) in a double four meets the axis of the other pair.

7. The Aberdonian travels every night from London to Aberdeen (570 miles) and Aberdeen to London, leaving each at 7.30 p.m. and arriving if running to time at 7 a.m. It stops for five minutes each at Peterborough (70 m. from London), Doncaster (150), York (210), Newcastle (320), Edinburgh (410), and Dundee (490). On this particular night, the train from London started 25 minutes late and because of fog was slowed down to 30 m.p.h. for 20 miles before Peterborough. The trains accelerate to and brake from their normal running speeds, which are constant wherever possible, steadily. If the other train was also held up for 15 minutes by a signal on the Forth Bridge, when they meet (*a*) which has taken longer and (*b*) which is further from London?

8. Six men leave a party in no fit condition to recognise their own property. If they were the only ones there, what is the chance that none of them takes away the right gown? (First principles, please!)

9. In the University of Dodeca, the following question was set in the Arithmetical Sopirt, Pt. III:

Divide 702XEX6 by 786.

It should be needless to observe that in this University all the teaching staff have six fingers on either hand. All the numbers are therefore expressed in powers of 12, which they quaintly call ten. The two numbers below ten are E and X, which correspond to our eleven and ten respectively. What is the correct solution?

10. "What's wrong with these dice?" I said suddenly.

"I've just rearranged them a bit," said Sam. "The chances of a 7 are better by a third, and I've changed as few other chances as possible."

What were the numbers on the two dice?

(*Solutions on page* 16.)

Saint Petersburg

Abel and Baker play under the following conditions: Abel is to toss a coin until it falls heads, at which time the game is to stop and Abel is to pay Baker 2^{n-1} chips, where n is the number of throws. How much should Baker stake on the game?

Aerodynamic Noise

By P. Martin

The problem of noise, particularly in connection with modern high-speed aviation, has become increasingly important in recent years. The intensity of sound produced by jet engines increases rapidly with the power output, so that for the large engines now coming into use, the problem is a major one. Apart from this technological importance, it is also of considerable fundamental interest.

The foundations of the science of sound were truly laid by Rayleigh and Stokes in the nineteenth century, who recognised two fundamental mechanisms for the conversion of kinetic energy into acoustic energy, as follows:—

(i) The mass in a fixed region of space may be caused to fluctuate, for example, by a sphere whose radius oscillates, or by a loud-speaker embedded in a very large baffle.

(ii) The momentum in a fixed region of space may be caused to fluctuate, or equivalently, the rate of mass flux across fixed surfaces may vary. This occurs when a solid object, say a tuning fork, vibrates after being struck.

Recently, Lighthill has shown that a third mechanism may in some circumstances be important. It is that

(iii) The *rates* of momentum flux across fixed surfaces may be caused to vary, as occurs in turbulent motion in the absence of solid boundaries.

Mathematically, it is well known that the difference between acoustic fields of the first and second types is the difference between the fields of an acoustic source and an acoustic dipole. The mechanism (ii) is far less efficient with regard to the total noise produced than is (i); for a purely radial motion of a sphere, the acoustic power output is 13 times what it would be for oscillations of the same amplitude of a rigid sphere. In the case of mechanism (iii), this "Stokes effect" is even more marked. Stokes himself showed that if the surface of a sphere vibrates in such a way that the volume and the position of the centroid remain fixed, the acoustic power is one-thousandth of what it would be for purely radial motion of the same amplitude. This last mechanism corresponds to an acoustic quadrupole. In a turbulent motion, it is the small part of the momentum flux which is not balanced by local reciprocal motions which then corresponds acoustically to a distribution of quadrupoles throughout the fluid.

Lighthill has shown that the amplitude of the quadrupole strength per unit volume is proportional to the square of a typical velocity U of the flow. The amplitude of the radiation field of a quadrupole is proportional to the quadrupole strength and also to the square of its frequency. In turbulent flow, a typical frequency is roughly proportional to the velocity U, so that the amplitude of the radiation field increases roughly as the fourth power of U. Hence the intensity increases as U^8.

Clearly, for the high speeds typical of modern aircraft, the intensity of this field can become very great. This is in accord with the observed very rapid increase of aerodynamic noise* with velocity, whether from aircraft flying at high speed or from large jet engines "warming up" on the ground. In the latter case, the high velocities occur near the orifice of the jet. I. Proudman has calculated in detail the sound produced by a region of isotropic turbulence, and confirms the general predictions of the theory as well as determining the magnitude of the noise. Lighthill has since shown that the presence of a large mean shearing motion in the fluid amplifies the intensity of the sound; he describes it as an "aerodynamic soundingboard." Near the orifice of a jet, where such large mean shearing motions occur, the sound produced is considerably more than it would be as a result of the turbulence alone.

But what of the other two mechanisms which, as has been suggested, might possibly be more efficient in producing noise? In the first place, for rigid structures, there are no fluctuations in the volume occupied by the body, so that no source field can exist. At a solid surface in contact with turbulent fluid, however, fluctuating stresses are set up between the boundary and the fluid. This represents a mechanism of type (ii), and corresponds acoustically to a distribution of dipoles on the surface. The amplitude of the radiation field of a dipole distribution is proportional to the dipole strength and to only the *first* power of the frequency. By considerations similar to those in the case of the quadrupole volume noise, we are led to a U^6 law for intensity of the surface noise. In view of the more rapid increase with velocity of the acoustic power output of the quadrupole field, it is clear that, for high enough speeds, this will be dominant. But it seems *a priori* possible for there to be a range of velocities in which the noise output is not negligible, and in which the surface noise, relatively more efficient at lower speeds, is more important. This proves to be so, and it has been shown

* The distinction between aerodynamic noise and purely engine noise must be kept in mind.

that for turbulent flow past a rigid, flat surface, the dipole noise dominates when the Mach number* is less than about unity.

Another aspect of the production of noise by turbulence lies in the effect of this on the initial turbulence. At low mach numbers, the energy of such a motion is dissipated by viscosity, but when the Mach number of the velocity fluctuations is large, the radiation of sound energy is an additional sink for the kinetic energy of the motion.

A great deal of research remains to be done on this topic and there is at present considerable experimental and theoretical investigation being carried out.

A Song . . .

Against Mathematicians

Of all lunatic professions which are practised on this earth
Mathematics is the craziest, and has been from its birth.
Take a look at its practitioners, examine each in turn,
And watch them going farther round the bend as more they learn:
  The proud arithmetician, who can contemplate infinities
  With crude familiarity, and not see what a sin it is,
  (Infinities of such a size, he calls a set equal if
  The greatest of their differences is small compared to aleph;)
The negligent dynamicist, Procrustes of equations,
Ignoring any higher power which foils his machinations;
If there is any man with more impiety than him, it
Is the unrepentant analyst, proceeding to the limit.
  There is a young geometer, aged 23 mod 40;
  His views on art are trivial, his views on life are naughty;
  The only scheme of government this student can envisage is
  To couple all constituents in independent syzygies.
Ye narrow minded bachelors, whose one joy is to figure!
Were you cut up and randomised, set down in utmost rigour,
Your singularities enclosed in everlasting cedar,
Who would lament your absence? . . .

But I leave that to the reader.

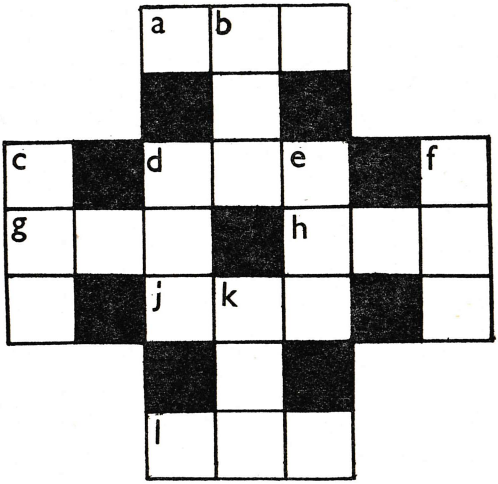
D. Handscomb.

---

* Mach number = ratio of fluid velocity to the velocity of sound in the undisturbed medium.

Cross Number Puzzle

By Pythagoras



Clues.—Capital letters refer to clues across, lower case letters to clues down.

$$\begin{aligned}
(D + G - L - 1)^2 &= (c + G + 2)^2 \\
&= A^2 + f^2 \\
&= d^2 + J^2 \\
&= (b + H)^2 + (e + 1)^2 \\
&= (2c)^2 + \left(\tfrac{1}{7}L\right)^2 \\
&= D^2 + \left(\tfrac{3}{2}k\right)^2 \\
&= (G + f)^2 + H^2
\end{aligned}$$

Word Problem. (*Solution on page* 17.)

From the four relations $ABA = A$, $BAB = B$, $AB = BA$, $AC = CA$ prove formally that $BC = CB$. (R. Penrose.)

Construction Problem. (*Solution on page* 17.)

Using only a circular coin of unit radius, one is allowed to construct circles of unit radius through any two given points which are not more than a distance of 2 apart. The problem is to construct a circle to touch another such circle at a given point. (R. Penrose.)

Logarithms to Base 1·024!

By N. M. Gibbins

In this article is set forth an unorthodox method of obtaining seven figure common logarithms. The apparatus consists of an algebraic inequality and a geometric one.

1. Let $$f(x) = \left(\frac{1}{2} + \frac{1}{x}\right) \log_e (1 + x).$$

Then $$(1 + x)\, f'(x) = -\frac{1}{x^2}(1 + x) \log_e (1 + x) + \frac{1}{2} + \frac{1}{x},$$

and by substituting the power series expansion for $\log_e (1 + x)$, we have

$$(1 + x)\, f'(x) = \frac{x}{2\cdot 3} - \frac{x^2}{3\cdot 4} + \frac{x^3}{4\cdot 5} - \ldots,$$

which is positive for $0 < x < 1$. Hence, if $0 < x < y < 1$,

$$\left(\frac{1}{2} + \frac{1}{x}\right) \log_e (1 + x) < \left(\frac{1}{2} + \frac{1}{y}\right) \log_e (1 + y),$$

so that $$\log_{1+y} (1 + x) < \frac{x(y + 2)}{y(x + 2)}. \qquad \ldots \quad \ldots \quad (1)$$

2. Now $$(1\cdot 024)^5 = 1\cdot 1258999 + \quad ,$$

and hence the inequality

$$(1\cdot 024)^5 < 1\cdot 1259 = 9/8\, (1\cdot 0008). \ldots \quad \ldots \quad (2)$$

Therefore $$\frac{(1\cdot 024)^{10}}{1\cdot 25} < \frac{81}{80}\, (1\cdot 0008)^2.$$

Taking logarithms to the base $1\cdot 024$ (denoted by Log) of this expression, and using the inequality (1), it becomes

$$10 - \text{Log } 1\cdot 25 < \frac{253}{3 \times 161} + \frac{2 \times 253}{3 \times 2501},$$
$$= 0\cdot 59125,$$
$$\text{Log } 1\cdot 25 > 9\cdot 40875.$$

But, since $1 \cdot 25 = 10 \cdot 2^{-3}$, and $1 \cdot 024 = 10^{-3} \cdot 2^{10}$, we have

$$10 \text{ Log } 2 - 3 \text{ Log } 10 = 1, \tag{3}$$

$$-3 \text{ Log } 2 + \text{Log } 10 > 9 \cdot 40875,$$

whence $$\text{Log } 10 > 97 \cdot 0875.$$

Inverting to obtain logarithms to the base 10 (denoted by log),

$$\log 1 \cdot 024 < 0 \cdot 0103, \tag{4}$$

and, substituting for $1 \cdot 024$ in powers of 10 and 2,

$$\log 2 < 0 \cdot 30103. \tag{5}$$

3. We need a lower bound for log 2, which can be found as follows:—

$$\text{Log } \frac{1 \cdot 28}{(1 \cdot 024)^{10}} < \text{Log } \frac{1 \cdot 28}{1 \cdot 26765},$$

$$< 0 \cdot 408815 \qquad \text{from (1).}$$

Since $1 \cdot 28 = 10^{-2} \cdot 2^7$,

$$7 \text{ Log } 2 - 2 \text{ Log } 10 < 10 \cdot 408815,$$

and this, together with (3), gives

$$\text{Log } 10 < 97 \cdot 08815,$$

As before, $$\log 1 \cdot 024 > 0 \cdot 0102999,$$

$$\log 2 > 0 \cdot 30102999.$$

Hence $$\log 2 = 0 \cdot 3010300. \tag{6}$$

4. Using this value, and the inequality (1), other results can be obtained readily. For example from (1) and (4),

$$\log \frac{81}{80} < \frac{253}{161 \times 3} \times 0 \cdot 0103,$$

$$< 0 \cdot 00539524,$$

and from (5), $$\log 80 < 1 \cdot 90309,$$

so that $$\log 3 = \frac{1}{4} \log 81 < 0 \cdot 47712131. \tag{7}$$

Similarly, $$\log \frac{50}{49} < \frac{253}{99 \times 3} \times 0{\cdot}0103,$$
$$< 0{\cdot}00877408.$$

Therefore $$\log \frac{100}{49} < 0{\cdot}30980408,$$
$$\log 49 > 1{\cdot}6909592,$$
$$\log 7 > 0{\cdot}84509796, \qquad (8)$$

and so on.

Solutions to Problems Drive

1. XOR, OQY are Simson's Lines of ACD, ABC so XPY is parallel to AB.

2. If there are n of them, and the remainder is r, and if x is the number before one stage and x' after, then
$$(1 - 1/n)\ (x - r) = x'.$$
So if $x = x'$, i.e. $x = -r\ (n - 1)$, the number is the same before and after each stage. Add n^{n+1} any number of times, and this is a general solution. For 3,1, the answer is 79.

3. (*a*) 21 (sum of natural numbers), (*b*) 63 (adding powers of 2), (*c*) 144 (alternate terms of Fibonacci Series and its own second difference series), (*d*) 38 (product of two primes).

4. A solution (not unique) is 0123456789 = ESROVNTWUF.

5. (*a*) $(4! - \sqrt{4})/(\sqrt{\cdot\dot{4}}) + 4$. (*b*) $4! + 4! + 4/\cdot\dot{4}$).
(*c*) $(4/\cdot\dot{4})^{\sqrt{4}} - 4$. (*d*) $4(4!) + (4/4)$. (*e*) $(\sqrt{4}/\cdot 4)! + \sqrt{(4/\cdot\dot{4})}$.

6. Let a, b, c, d and a', b', c' d' be lines and transversals. Then a, b define a solid containing ab', $a'b$, and c, d define a distinct solid containing cd', $c'd$. These solids meet in a plane in 4-space, so the four points are coplanar and their joins meet.

7. (*a*) The one from Aberdeen, (*b*) neither. (27 per cent. of competitors missed this.)

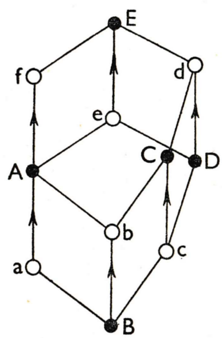
8. D.E. is $p\ (n) = n! - \binom{n}{1} p\ (n - 1) - \ldots \binom{n}{2} p\ (2) - 1$. p (1 to 6) are thus, 0, 1, 2, 9, 44, 265 and the chance is 53 : 144.

9. 786)702XEX6(XE19
6510
———
71XE
7096
———
115X
786
———
5946
5946
———

10. 123356, 124456. (223456, 123455 are excluded by "rearranged")

Solution of Construction Problem. (*Page* 13.)

In the diagram, the black spots represent intersections of circles and the white ones the centres of circles. The lines joining the spots are of unit length. If *a* and A are respectively the given circle and the given point, we can construct the figure in the following manner. We choose B arbitrarily on *a* and construct *b*, then C on *b*



to give *c*, and D (not too far from A) on *c* to give *d* and *e*. This determines E and we can construct *f*. The lines with arrows are parallel.

Solution to Word Problem. (*Page* 13.)

Written in full, this requires 17 steps. We have
$BC=B^2AC=B^2CA=B^2CA^3B^2=B^2A^3CB^2=ACB^2=CAB^2=CB$

Book Reviews

Geometrical Mechanics and de Broglie Waves. By J. L. SYNGE. (Cambridge University Press.) 25s.

Hamilton developed the theory of geometrical optics from Fermat's variational principle. This theory enables us to perform calculations which would be intractable in terms of the more correct Maxwell theory of wave optics. Professor Synge develops a relativistic geometrical mechanics from the principle of least action expressed in terms of Minkowski space-time. The set of three dimensional surfaces orthogonal to the extremals for given initial conditions are just a set of de Broglie waves. No notion of frequency or wave length has yet been introduced. To do so is shown to be equivalent to a primitive quantisation procedure. It is then possible to obtain the correct fine structure formula for the hydrogen atom solely from this formulation of geometrical mechanics. The book concludes with some interesting generalisations dealing in particular with the two body problem.

The chief interest in this book must lie in the clarity and generality with which the theory is presented. In this Professor Synge is generally successful though your reviewer found the simple illustrative examples inserted to illuminate general principles were at times more distracting than helpful.

J. C. P.

Introduction to Dynamics. By L. A. PARS. (Cambridge University Press.) 31s. 6d.

This book discusses motion in two dimensions and uses only elementary methods without recourse to Lagrange's equations or the more sophisticated apparatus of general dynamics. Consequently it should guide the tripos candidate through Part I successfully but no further—a short trip, some may think, for the financial outlay involved.

This is the price one must pay for luxury travel. Your reviewer is unable to agree with the implication expressed by Mr. Pars in his preface that hitherto no adequate book has existed which covers the ground but he can call to mind no book that covers it so carefully, with so much attention to clear and precise explanation of the ideas of elementary dynamics and with such a wealth of illustration. The earnest student of this book will find more than a handbook of problem solving techniques, he will discover the basic ideas of one of the most fascinating branches of mathematics.

Returning to its more practical advantages, it contains an excellent collection of examples and adopts a sensible attitude towards the use of vectors.

It is to be hoped that Mr. Pars will turn his attention to the more advanced branches of dynamics where there is still a great need for a modern book of this character.

J. C. P.

The Spirit of Applied Mathematics. By C. A. COULSON. (Oxford University Press.) 2s. 6d.

Prof. Coulson has devoted his Oxford inaugural to an apologia for applied mathematics. He sees his subject as an intellectual adventure in which we are seeking for an understanding of the form and structure of the physical universe. Consequently applied mathematics deals primarily with concepts and not with computation; its end is comprehension, not calculation. This enquiry must be undertaken as a creative activity and is subject to the canons of beauty and fitness. Applied mathematics has the advantage over its nearest neighbours—pure mathematics and experimental science—that it is a dynamic and not a static subject. Physical facts and mathematical theorems are unalterable, but our insight into the pattern of nature is continually developing.

The argument is expressed with Prof. Coulson's customary clarity and felicity. This lecture should be read by all who aspire to be applied mathematicians—and by all pure mathematicians also.

J. C. P.

An Introduction to Homotopy Theory (Cambridge Tract No. 43). By P. J. HILTON. (Cambridge University Press.) 15s.

The pace at which algebraic topology is developing to-day presents the student with a formidable task, which is not made any easier by the absence of suitable text-books in certain branches of the subject. In particular a book on homotopy theory is very much overdue, and Dr. Hilton has performed a valuable service in filling the gap.

The first four chapters of this tract are devoted to an exposition of the basic results of homotopy theory, and the next two deal with fibre spaces, the Hopf Invariant and Suspension Theorems. The final two chapters are of a different character from the rest of the book, being concerned with some advanced topics of more specialised interest. These are based mainly on the work of J. H. C. Whitehead on generalised complexes, and incorporate some of the author's own contributions to the subject. The whole treatment is highly algebraic and a close attention is paid to detail. The text is well written and easy to follow, though it requires careful reading.

The tract ends with a bibliography, which contains a very useful list of original papers. There is also a combined index and glossary which the reader will find most helpful. The printing and general lay-out are excellent and the University Press must be congratulated on procuding an attractive book.

M. J. A.

Methods of Algebraic Geometry (Vol. 3). By W. V. D. HODGE and D. PEDOE. (Cambridge University Press.) 40s.

This volume, which is devoted to birational geometry, concludes the authors' exposition of methods of algebraic geometry. With the needs of the classical geometer in mind, they again consider only groundfields without characteristic.

The first chapter contains a purely alebraic account of ideal theory. The next chapter is concerned with the arithmetic theory of varieties and deals with such topics as simple points and normal varieties. It is followed by an account of valuations, in the latter part of which particular attention is paid to the function field of an algebraic variety.

The theory of birational transformations is expounded in the first part of the long final chapter. The rest of the chapter consists of applications of this theory to problems involving the multiple points of an algebraic variety. The local uniformisation theorem is proved in sections 5–7, and the result applied in the last two sections to the reduction of the singularities of an algebraic surface. The account given is

essentially the "simplified" proof of Zariski. By making use of methods drawn from all three volumes this chapter provides an impressive demonstration of their scope.

With the completion of this work, the classical geometer has at his disposal an excellent introduction to modern methods and results. There is now no reason why the harmful division which has occasionally existed between the classical and modern schools should continue.

M. A. H.

An Analytical Calculus. By E. A. MAXWELL. (Cambridge University Press.) Volume I, 15s.; Volume II, 18s.; Volume III, 15s.

Dr. Maxwell's aim in writing these books is to present a complete course in calculus in such a way that the more exact treatments of analysis follow as a natural development. This task is a difficult one, and to be successful in it, required a very careful balance between the needs of rigour on the one hand and of directness in treatment on the other. The difficulties must be pointed out and not hidden by the rough-and-ready methods frequently encountered in elementary text-books. There must be a true logical development, clearly defined for the pupils coming to the subject for the first time. The treatment should be sound, without the distraction of attention to a standard of rigour which is out of keeping with a book of this nature.

The success with which this balance is struck is admirable. The treatment is elegant and stimulating, and the exposition clear. These books are invaluable to the student who wishes to either continue his studies in mathematics or to use the techniques of calculus directly in whatever application he chooses.

Volume I introduces first the *idea* of differentiation with the notions of function, limit and continuity. The evaluation of the differential coefficient of a function follows, with subsequent applications to dynamics and the properties of simple curves. Integration is treated in the same systematic way, with the basic ideas presented first and then the details of the various techniques and their applications. A number of examples are included on each topic, though probably not entirely sufficient for the pupil at this stage.

In Volume II are presented some of the more advanced parts of the theory, continuing about as far as Scholarship standard. The logarithm is defined as the integral of t^{-1}, from which its properties, and those of the exponential function are found. Some of the difficulties involved in Taylor's Theorem are pointed out; this section should serve as a warning to those who regard this theorem as "quite obvious." The treatment of the properties of curves is interesting and rather more satisfying in some respects than that usually given. There is a long chapter on complex numbers and the volume concludes with two short chapters on systematic integration and "infinite" integrals.

The third volume contains a treatment of functions of several variables. Partial differentiation and maxima and minima are discussed in some detail, and lead to a study of Jacobians and multiple integration. The work continues to be characterised by the maintenance of a reasonable standard of rigour. A good example is the geometrical discussion of multiple integration in polar and spherical coordinates without resort to "approximately equal infinitesimal elements." An instructive chapter on sketching curves of the type $f(x,y) = 0$ concludes the volume. There are again many examples, this time of Scholarship and University level.

The reviewer looks forward to reading the final volume in this series. These books amply answer the long felt need for a treatment of the calculus in its own right, paying attention to the difficulties but keeping the actual exposition clear and simple.

O. M. P.

Professional Opportunities in Mathematics. Arnold Buffum Chase Fund. (Mathematical Association of America.) 25c.

This is a report of a committee of the Mathematical Association of America, outlining the opportunities open to graduates in mathematics. Though the emphasis is naturally American, the subject-matter, with the possible exception of the section dealing with Federal Government posts, is sufficiently universal to give the work a wider interest.

The report is brief, yet fairly comprehensive. It deals with posts in statistics, industry, the teaching and actuarial professions, and in universities and the Government; advice is given concerning the type of University course which should be followed in each case. There is a bibliography giving a fairly wide selection of books and pamphlets, not all concerned with specific openings.

Undergraduates should find this report of use at all stages of a course in mathematics; it will be of particular value to someone who must choose between alternative courses, and who is not fully aware of the openings these courses will make available to him.

J. L. M.

Mathematical Association

President: PROFESSOR T. A. A. BROADBENT.

The Mathematical Association, which was founded in 1871 as the *Association for the Improvement of Geometrical Teacing,* aims not only at the promotion of its original object, but at bringing within its purview all branches of elementary mathematics.

The subscription to the Association is 21s. per annum: to encourage students, and those who have recently completed their training, the rules of the Association provide for junior membership for a limited period at an annual subscription of 10s. 6d. Full particulars can be had from The Mathematical Association, Gordon House, Gordon Square, London, W.C.1.

The *Mathematical Gazette* is the journal of the Association. It is published four times a year and deals with mathematical topics of general interest.
