Offer-Holders' Document (Mathematics)

Ben Laurense, Ming Yin (Maths Faculty Student Representatives);
Parth Shimpi, Neel Nanda (Cool Dudes)

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1 Introduction

The aim of this document is to give you an idea of what studying maths at Cambridge is like, and explain briefly what you'll actually be studying. University is a wholly different experience from school, and the transition can be a bit daunting - hopefully this document helps ease the transition. At the end, there is a section giving examples of maths-related things you can occupy yourselves with over this summer.

The *Centre for Mathematical Sciences*, or the *CMS*. This place is a home away from home for all mathematicians

1.0.1 A Note About Accuracy

The information in this document is correct to the best of our knowledge. However, there may be some (hopefully minor) mistakes or inaccuracies, all of which are our own. This document is endorsed by the Faculty of Mathematics.

1.0.2 STEP

STEP II and III are going ahead online, as I'm sure Cambridge has let you know. This is probably causing many of you a lot of stress, especially given most other exams have been cancelled. You're all in similar positions, and we all have online exams too so can empathise somewhat. Probably the best thing to do in this situation is try and form a routine, keep busy, and crack on with STEP preparation.

STEP is meant to be difficult at the best of times, but don't let yourself be put off if you're struggling - everyone is. You get better at STEP over time - it can feel fruitless, but practice will help build fluency

and the hard skills required. It's also worth noting that around one third of people who were accepted into Cambridge in recent years miss their STEP offers - struggling with STEP does not mean you won't get into Cambridge.

There are many resources available online for STEP preparation:

- The STEP Support Programme is a fantastic hub of resources, including modules to help you revise and practise certain topics.
- The official STEP website links a lot of useful resources, including past papers.
- Underground Maths is good revision of A-Level topics.
- If you search hard enough on the STEP threads of TheStudentRoom you can find grade boundaries for the more recent STEP papers.

If you have any questions about STEP, a good first point of contact is your chosen College's admissions email. Also, if you know anyone doing STEP, they're probably feeling a lot of what you are feeling right now, so it's worth reaching out.

1.0.3 Maths with Physics

For those of you who applied for maths with physics, it's worth noting that you can still switch to the only-maths option in the first term. You will have enough time to consult with current students and faculty before making an informed decision, so don't worry!

One thing that many students don't fully appreciate until they get to Cambridge is just how much theoretical physics there is in the Maths Tripos. For example, the mechanics and special relativity that you learn in first-year maths actually goes beyond the first-year content in physics. Having done STEP, you will have a stronger mathematical background than those taking Natural Sciences, and so the physics in first year of the Maths Tripos is taught with a stronger mathematical emphasis than in NatSci, which you may find more appealing. In third year, there are so many courses in the Maths Tripos that it's possible to specialise (almost) solely in theoretical physics. On the other hand, the main thing offered by physics in the Natural Sciences Tripos that is completely missing in the Maths Tripos is experimental physics — if you enjoy the practical side of physics, then maths with physics might be the better option for you than straight maths.

2 What Cambridge Maths is Like

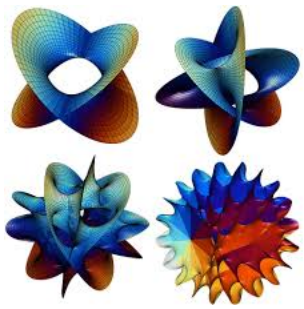
University maths is a large paradigm shift from A-Levels (and also not much like STEP), and first year (1A Maths) is designed to transition you from A-Levels to a university setting. First year does cover a chunk of A-Level content (since different A-Levels/qualifications covered different content). This section aims to give a flavour of what Cambridge maths is like.

2.1 What Maths Will I Learn?

2.1.1 Pure and Applied Maths

Maths is split very loosely into 'pure' and 'applied'. 'Pure' maths roughly means a *rigorous* focus on *abstract* concepts and objects and an exploration of their important properties. 'Applied' maths focuses on *developing tools and methods* for solving more real-world problems and using them *in context*. For example, in a pure setting, you might define differentiation and prove some important properties of it, like the chain rule; in an applied setting you might have to solve a physics problem based around a clever use of derivatives.

A nice way to phrase the distinction is: applied mathematicians are driven by the solution, all the tools are just *means* to get there. Pure mathematicians on the other hand care more about the *tools themselves*, what makes the machinery work and when it breaks.



Four Calabi-Yau threefolds, which look cool

2.1.2 What Courses Are Available?

In first year, everyone takes the same eight courses - four pure and four applied, each 24 lectures. In second and third year, you get to choose from a wide array of pure, applied, physics, and 'applicable' (loosely stats, decision and finance maths) courses. The official schedules can be found here.

- Numbers and Sets (pure, Michaelmas)
  This course covers two of the most fundamental concepts in maths: *arithmetic* and *sets* (collections of things). There's also a super cool section at the end about sizes of infinities! However this course is less about the destination and more about the journey - the journey here being 'learning to think like a mathematician' and 'becoming fluent with rigour and proof'

  Definition (Set)**.** A *set* is a collection of stuff, without regards to order. Elements in a set are only counted once. For example, if $a = 2, b = c = 1$, then $A = \{a, b, c\}$ has only two members. We write $x \in X$ if x is a member of the set X.

  Example. Common sets and the symbols used to denote them:

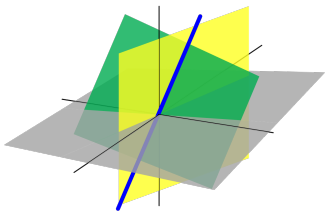
  - $\mathbb{N} = \{1, 2, 3, \cdots\}$ is the natural numbers
  - $\mathbb{N}_0 = \{0, 1, 2, \cdots\}$ is the natural numbers with 0
  - $\mathbb{Z} = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$ is the integers
  - $\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$ is the rational numbers
  - $\mathbb{R}$ is the real numbers

- Differential Equations (applied, Michaelmas)
  This is an introductory course to differential equations and gives you some basic tools to analyse them. Differential equations are fundamental to most of applied maths, physics, and engineering!

- Groups (pure, Michaelmas)
  This is a first course on *abstract algebra*, which is the study of *structure*. In this course you study groups, which are sets with a single well-behaved operation — these structures capture the idea of *symmetry* and can be found everywhere. The course equips you with a standard toolkit on how to look at algebraic objects — which you will keep using throughout your career as an algebraist. Dr Randal-Williams' lecture notes

- Vectors and Matrices (applied, Michaelmas)
  This is a first course in *linear algebra*, which is the study of the geometry of lines, planes and higher-dimensional equivalents (and by extension, matrices and systems of linear equations). Linear

algebra is super important and foundational to many fields (both pure and applied and also a lot of computing and physics!). This course is an applied perspective, with the goal of making you familiar with ideas that you will formalize in the pure Linear Algebra course next year. Prof Cowley's lecture notes



- Probability (pure, Lent)
  Probability is the mathematical study of uncertainty, and how to understand and quantify it. In our opinion, this is one of the most important tools to ever come out of mathematics. It's the basis of statistics, which is key to all kinds of areas of society. Vittoria Silvestri's lecture notes

- Vector Calculus (applied, Lent)
  Vector Calculus is the study of calculus in higher dimensions, and is the language in which a lot of physics (like electromagnetism and fluid dynamics) is written. You'll develop tools for working with derivatives and integrals in higher dimensions. Dr Anthony Ashton's lecture notes

- Analysis I (pure, Lent)
  This is a first course in analysis, the pure-mathematician name for calculus. This is a very formal and detailed course, the point is to start from some basic assumptions and carefully build your way up to a clear understanding of calculus. It's very detail-oriented, which is necessary because intuitions in Analysis can often be misleading. You'll have a lot of fun on example sheets (see below) finding counter-examples to 'obviously true' statements! Prof Körner's lecture notes

- Dynamics and Relativity (applied, Lent)
  This is a classical mechanics course, with a bit of special relativity at the end. The point is to explore a range of physical systems, and understand and model them. Prof David Tong's course webpage

In addition, there are two short courses that can be taken in the Easter term (Cambridge's name for the Summer term) of either first or second year:

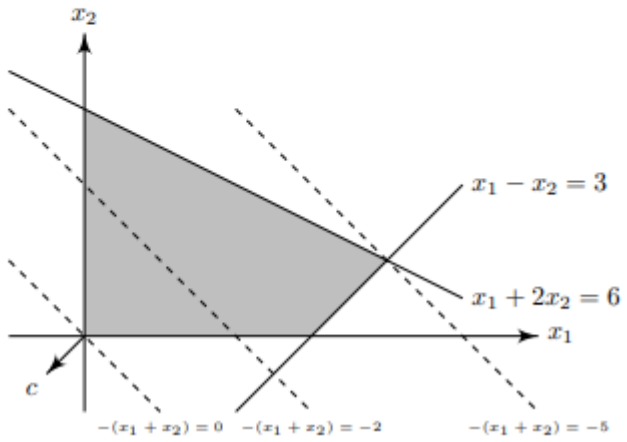
- Variational Principles (applied, Easter)
  Newtonian mechanics was reformulated in a beautiful way by Lagrange and Hamilton - instead of solving a system of differential equations, you could find *minima* of a special type of integral. This course develops tools for understanding and applying these special integrals

- Optimisation (applicable, Easter)
  Optimisation covers a bunch of decision maths problems about finding the maximum/minimum solution to an equation given constraints, such as linear programming

Example. We want to minimize $-(x_1 + x_2)$ subject to

$$\begin{aligned} x_1 + 2x_2 &\leq 6 \\ x_1 - x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Since we are lucky to have a 2D problem, we can draw this out.



2.1.3 Resources on the Web

As well as the lecture notes we've linked to above, here are a few websites with helpful resources for the Maths Tripos (these are all unofficial and not endorsed by the Maths Faculty):

Dexter's Notes — comprehensive set of lecture notes for undergraduate maths courses at Cambridge.

Gareth Taylor's personal website — resources compiled by a Cambridge supervisor.

Paul Minter's personal website — yet another Cambridge supervisor (& PhD student).

Many other resources can often be easily found by searching on Google and including the phrase "Mathematical Tripos".

2.2 What Happens in Lectures?

Cambridge maths is very strongly based on the lecture format as a means to teach, and this comprises almost all of your contact time with faculty. Cambridge maths lectures involve a lecturer speaking and writing on blackboards (or a visualizer), delivering content in a mostly linear fashion for $\approx$ 50 minutes.

The *Babbage Lecture Theatre* is where many first year maths lectures take place

Pure lectures roughly follow the 'lemma/lemma/theorem' format: an object/property we care about is defined and talked about for a while, and then *propositions* (mathematical statements) are proved about it - often, a sequence of smaller proofs (called *lemmas*) build up to a big, important, or fundamental result, called a *theorem*. If you want a better idea of what these lectures will feel like, the first few lectures of this MIT course on linear algebra are worth watching.

Applied lectures roughly involve the introduction of a new idea/method for solving a class of problems and then examples and exploration of this method (proofs are sometimes given, but more often we are just told the method works, perhaps with a sketch explanation to help with intuition). If you want a better idea of what these lectures will feel like, the first few lectures of this MIT lecture course on multivariable calculus and this MIT course on differential equations are worth watching.

Lecture attendance is not mandatory, and equally you can attend lectures for other courses you don't intend to take to exam (and even lectures from other degrees!). However, especially in first year, the Faculty encourages attending.

Some people prefer to study from printed notes (which many lecturers and students publish online) or books (the lecture courses have recommended reading, and between your College library, the University library, and the Faculty library you will be able to find just about anything).

Ben enjoys napping in the booths of the *Betty and Gordon Moore Library*, next to the CMS

2.3 What are Supervisions and Example Sheets?

In addition to lectures, you will have 'Example Sheets', which are sheets of questions based on the lecture content. There are four per course in first year, each covering ≈ six lectures of content. You hand these in to your supervisors (faculty members or PhD students) who go through your work with you in hour-long two-on-one sessions called 'supervisions'. Most of your working time will probably be devoted to example sheets, and people often meet with their supervision partners to discuss the sheets or help each other before the supervisions.

6. If n^2 is a multiple of 3, must n be a multiple of 3?

7. Show that, for every positive integer n, the number $3^{3n+4} + 7^{2n+1}$ is a multiple of 11.

8. Write down carefully (while not looking at your notes) a proof that there are infinitely many primes. By considering numbers of the form $4p_1p_2 \dots p_k - 1$, prove that there are infinitely many primes of the form $4n-1$. What would go wrong if we tried a similar proof to show that there are infinitely many primes of the form $4n+1$?

9. Prove that $2^{2^n} - 1$ has at least n distinct prime factors.

Excerpt from Numbers and Sets example sheet 1 (2019-2020)

Example sheets are *complementary* to the lectures - you will meet some ideas in lectures, but you will only truly understand them after playing with them in example sheets. Often, important results not proven in lectures are left for you to discover on your own through example sheet questions; so the sheets are pretty difficult by design and sometimes you will pick at a problem for more than an hour before the solution presents itself (and sometimes you won't be able to solve them at all). This is normal - there is no expectation to solve all the questions, and supervisors will sympathise if you had a deadline pile-up and couldn't do much on a sheet (many supervisors will have been through the undergraduate course themselves!). Also, you don't have to do the questions in order, so you can prioritise the more fun or interesting problems.

It's important that you spend enough time working on the problems individually, so you shouldn't see the

sheets as a 'group project', but it's incredibly useful, and you're strongly encouraged, to discuss the work with your supervision partner and other students - for example, asking for a hint if you get stuck or comparing your solution with a friend's to see if you took different approaches. Supervisors are also happy for you to email them any questions. Many students have study sessions with friends, and on Saturday afternoons several PhD students collect with tea and biscuits in the Centre for Mathematical Sciences for a 'Maths Cafe' where they will help undergraduates with their maths.

The Core of the Centre for Mathematical Sciences, where a lot of supervisions take place

The point of a supervision is not just to go through an example sheet, it is an opportunity to ask your supervisors about any parts of the course you're finding difficult. Therefore it's a good idea to chat to your supervision partner before a supervision and figure out which parts of the example sheet and which topics you'd like to focus on. Most importantly, **if you don't understand something, ask for help**!

2.4 Typical Timetable and Workload

In first year you'll have 2 lectures a day except Sundays (but including Saturdays!) and roughly 2 supervisions per week. Compared to A-Level, the work is certainly more intense, and you should expect to have to work harder. The faculty advises people to work between 42-48 hours a week, including lectures and supervisions, but this should be thought of as a VERY rough estimate - the courses are structured such that there's always more maths you could be doing, and how difficult you find certain topics will vary greatly. An extremely important skill to develop is balancing work with with hobbies, friends and relaxation, and feeling able to not complete much of an example sheet if you're feeling overworked.

It's highly recommended to spend the first two or three weeks of the first term (which are very relaxed maths-wise) paying attention to how you learn best, having a good routine, and your time-management. Most people just trust their intuitions about how they learn and go with the flow with regards to routine and time management, but these are skills that can be developed and improved. Optimising them early will keep you stay on top of work (and life in general). Additionally, people have different learning styles (which vary between courses and over time), and (we cannot emphasise this enough) it REALLY pays to experiment and find what works best for you.

At the beginning of term, you'll meet with your Director of Studies who will assign you supervisors and supervision partners. Term really kicks in in the second or third week when supervisions start!

2.5 Structure of First Year

Michaelmas term is typically devoted just to getting used to Cambridge - you made it here, rejoice! Freshers' week is a fun way to make friends and join some of the numerous societies. By the end of the

term you will have settled in, having explored the courses and societies the university has to offer, and discovered the routine that works best for you. Also, many summer internship applications close before the Christmas holidays are over, so make sure you're on top of that if you're career-minded (although having internships in first year is not the norm).

The Christmas holidays and early Lent is a good time to start looking for things to do over the summer, if you want to. Internship applications usually have specified deadlines, but some people find something late into Easter term. Don't worry! There are also summer maths projects both in the faculty and elsewhere, but they are usually restricted to second and third years. However, you can email supervisors/faculty members to look for more personalized projects in areas of maths you enjoy.

Easter term is mostly spent consolidating all the knowledge you have gathered in the year, as well as maybe attending Optimisation and Variational Principles. You have an entire term to prepare for exams, so we recommend not worrying too much about exams in the first two terms, instead using them to understand and appreciate the mathematics you are learning and have fun with the ideas.

3 Extra-Curricular Maths

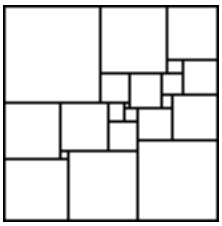
There are a bunch of maths societies in Cambridge which have regular speaker events, socials, and maths events.

- The Archimedeans

  "The Archimedeans is the university-wide mathematics society and is also the biggest mathematics society in Cambridge, hosting talks and events for the community. Distinguished speakers from within and outside Cambridge are invited to deliver talks on a variety of areas of mathematics, and periodic non-mathematical fun events are organized (watch out for Pizza and Board games nights!) You can also watch the talks on YouTube!"

  —Parth Shimpi, Archimedeans President

- The Trinity Mathematical Society (TMS)



The smallest possible **simple** squared square, logo of the TMS

*"Although the society is centered around Trinity College, we are very happy to be open to the entire university (no matter the subject or college). We organize a substantial portion of the over 20 maths talks for undergraduates in Cambridge each term (**significantly** higher than in many other universities in the world) and an annual symposium for talks by Ph.D. students. Finally, we do like to have some fun by organizing pub quizzes, garden parties, dinners, and other social events. Hopefully, we'll see many of you asking tough questions at our talks in the future."*

—Misha Schmalian, TMS President

- The Adams Society, the maths society of St. John's College
- The Emmy Noether Society, which aims to promote women in mathematics (their events are open to all though!)
- The Ethics in Mathematics Society

4 Maths to do This Summer

4.1 Maths to Revise

Having just completed STEP, you'll be in a good position to jump straight into university maths. The faculty will send out a short workbook in September with a bunch of easy questions, which you should make sure you're happy with. Make sure you're reasonably confident with A level core maths, but the first few lectures will go over important bits in case you've forgotten anything.

4.2 Fun/Relevant Summer Maths

There are many fun maths-related things you can do over the summer if you're so inclined.

Transitioning to university maths:

- Oxford's guide on how undergraduates do maths is very comprehensive and definitely worth a read
- How to Study for a Maths Degree by Lara Alcock
- How to Think Like a Mathematician by Kevin Houston
- An Introduction to Mathematical Reasoning by Peter J. Eccles

Don't ignore these resources! Taking the time to better understand what university-level maths is like will pay off massively once you start university. You don't necessarily have to buy a personal copy - you may be able to get a copy of one of these books online, or have a look in your local library once the lockdown eases.

Cambridge also has a nice reading list here.

Examples of University Maths:

- MIT OpenCourseWare Youtube Channel
- Any of the course notes we've linked above

Other fun maths:

- 3Blue1Brown Youtube Channel — great for building intuition and visualising difficult concepts
- Mathologer Youtube Channel — introduces lots of fun concepts to play around with
- Maths Olympiad papers from various countries, such as the UK and the US Olympiads. Especially good are the AoPS forums dedicated to the Olympiads on which a lot of cool maths is discussed
- Khan Academy — has some very well-designed resources for accessible advanced maths topics